NAG Toolbox for MATLAB

s19aa

1 Purpose

s19aa returns a value for the Kelvin function $\operatorname{ber} x$ via the function name.

2 Syntax

```
[result, ifail] = s19aa(x)
```

3 Description

s19aa evaluates an approximation to the Kelvin function $\operatorname{ber} x$.

Note: $\operatorname{ber}(-x) = \operatorname{ber} x$, so the approximation need only consider $x \geq 0.0$.

The function is based on several Chebyshev expansions:

For $0 \leq x \leq 5$,

$$\operatorname{ber} x = \sum_{r=0}{}' a_r T_r(t), \qquad \text{with } t = 2\left(\frac{x}{5}\right)^4 - 1.$$

For $x > 5$,

$$\begin{aligned}\operatorname{ber} x = \; & \frac{e^{x/\sqrt{2}}}{\sqrt{2\pi x}}\left[\left(1 + \frac{1}{x}a(t)\right)\cos\alpha + \frac{1}{x}b(t)\sin\alpha\right] \\ & + \frac{e^{-x/\sqrt{2}}}{\sqrt{2\pi x}}\left[\left(1 + \frac{1}{x}c(t)\right)\sin\beta + \frac{1}{x}d(t)\cos\beta\right],\end{aligned}$$

where $\alpha = \frac{x}{\sqrt{2}} - \frac{\pi}{8}$, $\beta = \frac{x}{\sqrt{2}} + \frac{\pi}{8}$,

and $a(t)$, $b(t)$, $c(t)$, and $d(t)$ are expansions in the variable $t = \frac{10}{x} - 1$.

When x is sufficiently close to zero, the result is set directly to $\operatorname{ber} 0 = 1.0$.

For large x, there is a danger of the result being totally inaccurate, as the error amplification factor grows in an essentially exponential manner; therefore the function must fail.

4 References

Abramowitz M and Stegun I A 1972 *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

5.1 Compulsory Input Parameters

1: **x – double scalar**

The argument x of the function.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **result – double scalar**

The result of the function.

2: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, ABS(**x**) is too large for an accurate result to be returned. On soft failure, the function returns zero.

7 Accuracy

Since the function is oscillatory, the absolute error rather than the relative error is important. Let E be the absolute error in the result and δ be the relative error in the argument. If δ is somewhat larger than the ***machine precision***, then we have:

$$E \simeq \left| \frac{x}{\sqrt{2}} (\mathrm{ber}_1 x + \mathrm{bei}_1 x) \right| \delta$$

(provided E is within machine bounds).

For small x the error amplification is insignificant and thus the absolute error is effectively bounded by the ***machine precision***.

For medium and large x, the error behaviour is oscillatory and its amplitude grows like $\sqrt{\frac{x}{2\pi}} e^{x/\sqrt{2}}$. Therefore it is not possible to calculate the function with any accuracy when $\sqrt{x} e^{x/\sqrt{2}} > \frac{\sqrt{2\pi}}{\delta}$. Note that this value of x is much smaller than the minimum value of x for which the function overflows.

8 Further Comments

None.

9 Example

```
x = 0.1;
[result, ifail] = s19aa(x)
```

```
result =
    1.0000
ifail =
          0
```